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# Non-planar string networks on tori

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Abstract: Type-II strings in $D = 5$ contain particle-like 1/8 supersymmetric BPS states. In this note we give a string-network representation of such states by considering (periodic) non-planar $(p, q, r)$-string networks of eight dimensional type II string theory on $T^3$. We obtain the BPS mass formula of such states, in terms of charges and generating-vectors of the torus, and show its invariance under an $\mathrm{SL}(3, \mathbb{Z}) \times \mathrm{SL}(3, \mathbb{Z})$ group of transformations. Results are then generalized to string-networks associated with the $\mathrm{SL}(5, \mathbb{Z})$ $U$-duality in seven dimensions. We also discuss reinterpretation of the above ($D = 5$) mass formula in terms of BPS states in world-volume theories of $U2$-branes in $D = 8$.

Keywords: Superstrings and Heterotic Strings, D-branes, String Duality, Superstring Vacua.

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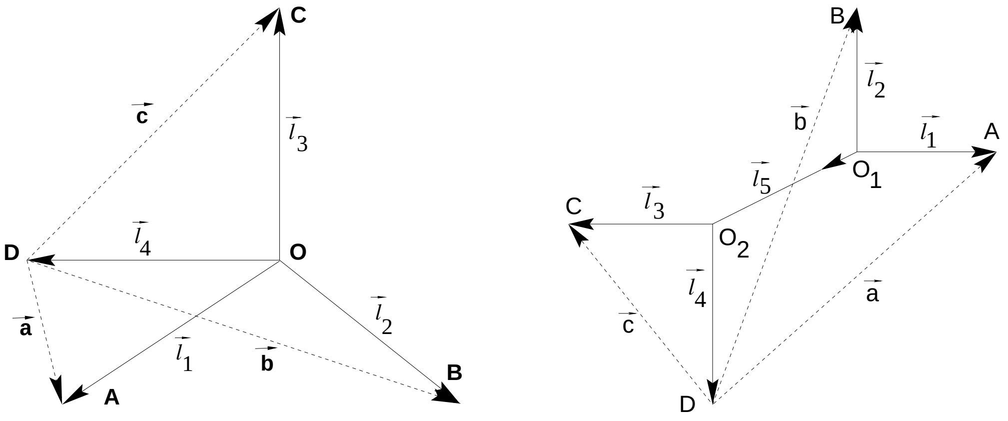
Classification of type-II BPS states with different supersymmetries have been discussed in several papers [1, 2]. They form $U$-duality multiplets of stings in various dimensions. The corresponding black hole entropies are also given by duality invariant expressions. However, its statistical understanding requires the degeneracy of states. It has been suggested [3] that string networks [3, 4, 5] may provide the degeneracy of such states, when considered on tori [3].

Network configurations have also been an important topic of discussion from various other angles in type-II string theory [6, 7, 8], $N = 4$ gauge theories [9, 10, 11] and non-commutative geometry [12]. In string theory, they provide 1/4, 1/8 and other lower supersymmetric states [3]–[8]. Moreover type-IIB planar string networks which end on D3-branes [9, 10] represent the 1/4 BPS dyonic states of $N = 4$ gauge theories in four dimensions [11]. Network-like configurations, have also appeared in other supersymmetric field theories [13], but are likely to have connections with those mentioned above. Recently, extension of network configurations to strings carrying 1-form electric charges (per unit length) and currents was also presented [5]. These can be of interest from the point of view of cosmological applications [14].

In this paper we study the application of string networks to $D = 5$ BPS states. It is known that one can have 1/8 supersymmetric particle-like states in $D = 5$ [2]. We give a string network representation of such states, by compactifying periodic $D = 8$ non-planar networks [4] of an SL$(3, \mathbb{Z})$-multiplet of type-II strings on $T^3$. We also generalize the results to the SL$(5, \mathbb{Z})$ U-duality in seven dimensions.

Our exercise can also be used to write down mass formula of 1/8 BPS states, in certain world-volume theories of 2-branes. These branes are themselves identified as $U$-duality branes [15] obtained from toroidal compactification of ten-dimensional branes. For the eight dimensional case, one notices that there exist 2-branes which are invariant under the SL$(3, \mathbb{Z})$ part of $U$-duality (for the purpose of this paper, SL$(3, \mathbb{Z})$ is the only relevant part of $U$-duality in eight dimensions). Masses of states, which can be identified as a string-junction ending on these branes can then be found from the above exercise. Such an analysis for D3-branes has been performed in great detail, including for the case of non-abelian gauge groups etc. [9, 10, 11]. We expect that, same should be possible for these $U$2-branes as well.

We now start by describing the periodic non-planar $(p, q, r)$ string networks of our interest, built out of basic structures as shown in figures-1. For convenience, we first consider the string networks, whose basic building blocks are 4-string junctions as in figure $1a$. The existence of such a junction can be seen from the general structure of non-planar string networks of [4]. In the construction of [4], the basic building blocks of the networks are 3-string junctions whose 3-prongs lie in a specific two dimensional plane in a three dimensional space, now identified as $T^3$. However, different junctions, including the adjacent ones can have their 3-prongs in different two dimensional planes, giving them a non-planar form (see figure $1b$). Then by




(a) (b)

**Figure 1:** $(a)$ A 4-string junction. $(b)$ A non-planar 4-prong.

shrinking the length of the intermediate links of such adjacent junctions, which is a free parameter in these BPS constructions, one gets a 4-string junction. Such objects have also been studied in [16].

A periodic structure of such 4-string junctions can then be constructed as a three dimensional generalization of the string network lattice in [3]. However one now needs four strings with SL(3) charges: $(p_I, q_I, r_I)$, $I = 1, 2, 3, 4$ to construct such 4-string junctions. By fixing the lengths of these string-links to $l_I$, $(I = 1, 2, 3, 4)$, and by imposing charge conservation condition on junctions: $\sum_{I=1}^{4} p_I = \sum_{I=1}^{4} q_I = \sum_{I=1}^{4} r_I = 0$, one obtains a periodic lattice. Although we do not present a pictorial representation of such 3-D periodic networks, their existence is guaranteed from the existence of the three dimensional lattice vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ given below in terms of the 'link-vectors' $\vec{l}_I$. These link-vectors themselves are given by the lengths of the prongs mentioned above and their orientation is given as in [4] in order to preserve 1/8 supersymemtry. More precisely, these orientations for a string with SL$(3, \mathbb{Z})$ quantum numbers $(P_I)_i \equiv (p_I, q_I, r_I)$ are given in terms of components $(X_I)_a$, $(a = 1, 2, 3)$ of a vector in real space (now identified as $T^3$):

$$\vec{V_I} = (X_I)_a \hat{e}_a \,, \qquad (a = 1, 2, 3)\,, \tag{1}$$

with $(X_I)_a$ given by:

$$(X_I)_a = (\lambda^{-1})_{ai} (P_I)_i \,. \tag{2}$$

'$\hat{e}_a$' in this paper always denotes orthogonal set of unit vectors in $T^3$, although its index '$a$' is chosen to be same as that of an internal SO(3) vector. $\lambda^{-1}$ in the above equation denotes the vielbein corresponding to the SL(3)/ SO(3) moduli:

$$G = \begin{pmatrix} g + a^T a e^{-\phi} & e^{-\phi} a^T \\ a e^{-\phi} & e^{-\phi} \end{pmatrix}, \tag{3}$$

with $g$ being a $2 \times 2$ matrix:

$$g = \begin{pmatrix} e^{(\phi+\alpha)} + \chi^2 e^{-\alpha} & e^{-\alpha}\chi \\ \chi e^{-\alpha} & e^{-\alpha} \end{pmatrix}. \tag{4}$$

Then one has

$$\lambda^{-1} = \begin{pmatrix} e^{-(\phi+\alpha)/2} & -\chi e^{-(\phi+\alpha)/2} & -e^{-(\phi+\alpha)/2}a_1 + \chi e^{-(\phi+\alpha)/2}a_2 \\ 0 & e^{\alpha/2} & -e^{\alpha/2}a_2 \\ 0 & 0 & e^{\phi/2} \end{pmatrix}. \tag{5}$$

To summarize these definitions, $P_I$'s defined above are internal $\mathrm{SL}(3,\mathbb{Z})$ vectors associated with the $I$'th string, whereas $X_I$'s are internal SO(3) vectors constructed by contracting $P_I$'s with the vielbein $\lambda$. This SO(3) is the maximal compact subgroup of SL(3). Finally $\vec{V}_I$'s in eq. (1) are vectors in $T^3$, due to their dependence on unit vector $\hat{e}_a$. Identification of its components with those of $X_I$'s in eqs. (1) and (2) is a property of the string networks, as the spatial and internal orientations of the links in a network are always aligned in a specific manner.

Major exercise now is to start from the expression of the mass associated with the above 4-string junction defining the unit cell[1] of a periodic network lattice and to show that these can be rewritten in terms of three independent $\mathrm{SL}(3,\mathbb{Z})$ charges $(P_I)_i \equiv (p_I, q_I, r_I)$, $(I = 1,2,3)$, $\mathrm{SL}(3)/\mathrm{SO}(3)$ moduli $G$, and three-dimensional vectors: $\vec{a}, \vec{b}, \vec{c}$ defined in terms of the lengths $l_I$'s of the four legs of the string junction as well as the unit vectors along these legs, $\hat{n}_I \equiv \vec{V}_I/|V_I|$:

$$\vec{a} = \vec{l}_1 - \vec{l}_4\,, \qquad \vec{b} = \vec{l}_2 - \vec{l}_4\,, \qquad \vec{c} = \vec{l}_3 - \vec{l}_4\,. \tag{6}$$

In fact, as we will see below various combination of $(\vec{a}, \vec{b}, \vec{c})$ provide additional moduli in the lower dimensional theory, after quantum numbers $(p_4, q_4, r_4)$ are eliminated in favor of the remaining ones, using charge conservation conditions.

Technical non-triviality of our exercise, with respect to the one performed in [3] for the planar network, is in dealing with the 3-dimensional problem in our case, compared to the 2-dimensional one in [3]. To perform this exercise explicitly, we first consider the case when $\mathrm{SL}(3)/\mathrm{SO}(3)$ moduli, $G$, have a diagonal form. It will be observed that the final expression that we derive, easily generalize to the most general moduli as well. For the diagonal case, $G$ has a form: $G = \mathrm{diag}(e^{\phi+\alpha}, e^{-\alpha}, e^{-\phi})$. Moreover for this case, the string tension is given as: $T_I = |X_I| = [e^{-(\phi+\alpha)}p_I^2 + e^{\alpha}q_I^2 + e^{\phi}r_I^2]^{1/2}$, $(I = 1,2,3,4)$.

We now use the above expressions to compute the mass of the BPS state, given by the string network configuration built by the above 4-string junctions. It is given by

$$m_{BPS}^2 = (l_1T_1 + l_2T_2 + l_3T_3 + l_4T_4)^2\,. \tag{7}$$

[1] See [3] for more details in the planar case.

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Now, to eliminate the lengths of the link-vectors of the strings in favor of the generating vectors $\vec{a}, \vec{b}, \vec{c}$, we use expressions of various scalar and vector combinations formed from these by taking their dots and cross products: $a^2 = l_1^2 + l_4^2 - 2\vec{l}_1 \cdot \vec{l}_4$, $\vec{a} \cdot \vec{b} = \vec{l}_1 \cdot \vec{l}_2 - \vec{l}_1 \cdot \vec{l}_4 - \vec{l}_2 \cdot \vec{l}_4 + l_4^2$, $\vec{a} \times \vec{b} = \vec{l}_1 \times \vec{l}_2 - \vec{l}_1 \times \vec{l}_4 - \vec{l}_4 \times \vec{l}_2$ etc. Moreover, these expressions can be rewritten in terms of quantum numbers $(p_I, q_I, r_I)$, $(I = 1, 2, 3)$, moduli fields $(\phi,\ \alpha)$, lengths of the links $l_I$, and their string-tensions $T_I$, by using relations:

$$\begin{aligned}
\vec{l}_1 \times \vec{l}_2 = \frac{l_1 l_2}{T_1 T_2} &\left[\hat{e}_1 (q_1 r_2 - q_2 r_1) e^{(\phi+\alpha)/2} + \right. \\
&\left. + \hat{e}_2 (r_1 p_2 - r_2 p_1) e^{-\alpha/2} + \hat{e}_3 (p_1 q_2 - p_2 q_1) e^{-\phi/2} \right], \qquad (8)
\end{aligned}$$

and two other expressions obtained by taking cyclic permutations in indices $(1, 2, 3)$ and $(p, q, r)$. Similarly,

$$\begin{aligned}
\vec{l}_1 \times \vec{l}_4 = -\frac{l_1 l_4}{T_1 T_4} &\left[\hat{e}_1 (q_1 (r_2 + r_3) - (q_2 + q_3) r_1) e^{(\phi+\alpha)/2} + \right. \\
&+ \hat{e}_2 (r_1 (p_2 + p_3) - (r_2 + r_3) p_1) e^{-\alpha/2} + \\
&\left. + \hat{e}_3 (p_1 (q_2 + q_3) - (p_2 + p_3) q_1) e^{-\phi/2} \right], \qquad (9)
\end{aligned}$$

and again two others obtained by the above cyclic permutations.

With the help of above expressions, and after some algebra, one can show that the mass of the BPS state, after $T^3$ compactification can be written as:

$$\begin{aligned}
m_{BPS}^2 = &\Big[ (\vec{V}_1 \cdot \vec{V}_1) a^2 + (\vec{V}_2 \cdot \vec{V}_2) b^2 + (\vec{V}_3 \cdot \vec{V}_3) c^2 + \\
&+ 2 (\vec{V}_1 \cdot \vec{V}_2)(\vec{a} \cdot \vec{b}) + 2 (\vec{V}_1 \cdot \vec{V}_3)(\vec{a} \cdot \vec{c}) + 2 (\vec{V}_2 \cdot \vec{V}_3)(\vec{b} \cdot \vec{c}) \Big] + \\
&+ 2 \Big[ (\vec{a} \times \vec{b}) \cdot (\vec{V}_1 \times \vec{V}_2) + (\vec{a} \times \vec{c}) \cdot (\vec{V}_1 \times \vec{V}_3) + (\vec{b} \times \vec{c}) \cdot (\vec{V}_2 \times \vec{V}_3) \Big] \\
\equiv\ & m_1^2 + m_2^2 \,, \qquad (10)
\end{aligned}$$

where $m_1^2$ and $m_2^2$ correspond to the terms in the two square brackets in eq. (10). This equation is one of the main result of this paper. It gives the BPS mass in terms of nine integers $(p_I, q_I, r_I)$'s, moduli $(\phi, \alpha)$ (through their appearance in $\vec{V}_I$), as well new set of moduli formed out of $\vec{a}, \vec{b}, \vec{c}$. The generalization of the result, to the case when the full set of $\mathrm{SL}(3)/\mathrm{SO}(3)$ moduli are turned on, is straight-forward. In that case, mass formula remains same as (10). However $\vec{V}_I$'s and $X_I$'s involve general $\mathrm{SL}(3)/\mathrm{SO}(3)$ moduli through their dependence on the vielbein in eq. (5).

We now show that the above mass formula has an $\mathrm{SL}(3, \mathbb{Z})_U \times \mathrm{SL}(3, \mathbb{Z})_u$ symmetry. The first $\mathrm{SL}(3, \mathbb{Z})$ is essentially the $U$-duality symmetry of type-II strings in eight dimensions. The second $\mathrm{SL}(3, \mathbb{Z})$ comes from the compactification of the network on $T^3$.

We first show the $\mathrm{SL}(3, \mathbb{Z}) \times \mathrm{SL}(3, \mathbb{Z})$ invariance of the terms in the first square bracket in eq. (10), identified as $m_1^2$. These terms can be rewritten as:

$$m_1^2 = P^T M P \,, \qquad (11)$$

where $P$ is $9 \times 1$ column vector with entries:

$$P = \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}, \tag{12}$$

and $M$ is a matrix:

$$M = \begin{pmatrix} a^2 G^{-1} & (\vec{a}\cdot\vec{b})G^{-1} & (\vec{a}\cdot\vec{c})G^{-1} \\ (\vec{b}\cdot\vec{a})G^{-1} & b^2 G^{-1} & (\vec{b}\cdot\vec{c})G^{-1} \\ (\vec{c}\cdot\vec{a})G^{-1} & (\vec{c}\cdot\vec{b})G^{-1} & c^2 G^{-1} \end{pmatrix}. \tag{13}$$

One can then write down the action of two SL(3)'s mentioned above on charges and moduli, including the ones constructed out of vectors $\vec{a}, \vec{b}, \vec{c}$. $\mathrm{SL}(3)_U$ has the identical action as in eight dimensions. This leaves any $T^3$ vectors such as $\vec{a}, \vec{b}, \vec{c}$ etc. invariant and acts on $P$ through a diagonal action on $P_I$'s as:

$$P_I \to \Lambda_U P_I\,, \qquad G^{-1} \to \Lambda_U^{-1T} G^{-1} \Lambda_U^{-1}\,. \tag{14}$$

with $\Lambda_U$ being $\mathrm{SL}(3,\mathbb{Z})$ matrices. Second symmetry, namely $\mathrm{SL}(3)_u$ acts on $P$ as:

$$\begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} \to \Lambda_u \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}, \tag{15}$$

namely, it mixes the indices $(1,2,3)$ associated with SL(3) charges $(p,q,r)$ of various strings among themselves. In addition, one has

$$\vec{A} \equiv \begin{pmatrix} \vec{a} \\ \vec{b} \\ \vec{c} \end{pmatrix} \to \Lambda_u^{-1T} \begin{pmatrix} \vec{a} \\ \vec{b} \\ \vec{c} \end{pmatrix}. \tag{16}$$

Due to the action of the symmetry group defined above, $9 \times 9$ moduli matrix in the compactified theory ($M$), transforms under SL(3)'s as $M \to (\Lambda_U^{-1T} \otimes I_3) M (\Lambda_U^{-1} \otimes I_3)$, $M \to (I_3 \otimes \Lambda_u^{-1T}) M (I_3 \otimes \Lambda_u^{-1})$.

We have therfore shown an explicit $\mathrm{SL}(3,\mathbb{Z})_U \times \mathrm{SL}(3,\mathbb{Z})_u$ invariance of the first part of the BPS mass formula (11). We also observe that by factoring out the volume of the polyhedron formed out of vectors $(\vec{a}, \vec{b}, \vec{c})$ from the matrix $M$ in (13), it can be identified with a matrix parameterizing $[\mathrm{SL}(3/\,\mathrm{SO}(3)] \times [\mathrm{SL}(3)/\,\mathrm{SO}(3)]$ moduli.

We now see that the terms in the second square bracket of (10), namely $m_2^2$, are also invariant under the transformations (14), (15) and (16). We first consider $\mathrm{SL}(3)_U$, after writing vectors $\vec{V}_I$'s as: $\vec{V}_I = T_I \hat{n}_I$, $(I = 1,2,3)$, with $T_I$ being the tensions of the strings and $\hat{n}_I$ being the unit vectors along them. $\mathrm{SL}(3,\mathbb{Z})_U$ invariance of $m_2^2$ then follows from the fact that it acts on various quantities inside second square bracket in (10) only through terms in the expressions of string tensions.

$SL(3,\mathbb{Z})_u$ symmetry of $m_2^2$ is also clear by noticing that although $\vec{V}_I$'s are spatial (or $T^3$) vectors, they transform under $SL(3,\mathbb{Z})_u$ due to its action on quantum numbers $p_I, q_I, r_I$'s in a similar manner as $P_I$'s mentioned above in (15). Then, using the definition $\vec{A}$ as in (16), the invariance of $m_2^2$ can be seen by writing it as:

$$m_2^2 = (\vec{A}_I \times \vec{A}_J) \cdot (\vec{V}_I \times \vec{V}_J)\,. \tag{17}$$

We have therefore shown an $SL(3,\mathbb{Z}) \times SL(3,\mathbb{Z})$ invariance of the mass formula obtained from a periodic network of 4-string junctions in eight dimensions.

A similar analysis goes through for more general periodic string-networks constructed out of 4-prong structures as shown in figure 1*b*. One now has lattice vectors defined as:

$$\vec{a} = \vec{l}_1 - (\vec{l}_4 + \vec{l}_5)\,, \qquad \vec{b} = \vec{l}_2 - (\vec{l}_4 + \vec{l}_5)\,, \qquad \vec{c} = \vec{l}_3 - \vec{l}_4\,. \tag{18}$$

The mass of the 1/8 supersymmetric BPS state associated with the compactified string network is now given by the expression:

$$m_{BPS}^2 = \left( \sum_{i=1}^{5} l_I T_I \right)^2 , \tag{19}$$

with lengths and tensions now being associated with the string-links in figure 1*b*. This expression, after similar algebra as above, can now be written as:

$$m_{BPS}^2 = P^T M P + \sum_{I=1}^{5} (\vec{l}_I \times \vec{l}_J).(\vec{V}_I \times \vec{V}_J). \tag{20}$$

Then using the definitions of the lattice vectors in (18), and charge-conservations on vertices $O_1$ and $O_2$ in figure 1*b*, one can show that the final 1/8 BPS mass formula is once again given by eqs. (10) and (11) and (17).

We also obtain the $SL(2,\mathbb{Z}) \times SL(2,\mathbb{Z})$ invariant formula of [3] by turning off appropriate moduli and charges. For example, when only nonzero SL(3) charges are: $(p_I, q_I)$, $(I = 1, 2)$, then by setting $\phi = a_1 = a_2 = 0$ in (10) one reproduces exactly the same expression as in [3]. This can be seen from the form of $m_1^2$ in (11), which reduces to the first term in [3, eq. (17)] in these limits. Moreover, for $m_2^2$ only one of the term in the second square bracket in (10) is nonzero and gives precisely the second term in [3, eq. (17)].

We now comment on the connection of these results with $U$-duality in $D = 5$. The full $U$-duality symmetry in $D = 5$ is $E_{6(6)}$ and gauge charges are in its **27**-dimensional representation. $E_{6(6)}$ however has an SL(6) subgroup whose origin can be seen from the interpretation of the $D = 5$ theory as $T^5$ compactified M-theory. This SL(6), in turn, has an $SL(3) \times SL(3)$ subgroup which can be identified with $SL(3)_U \times SL(3)_u$ mentioned above. Nine charges represented by $p_I, q_I, r_I$, $(I = 1, 2, 3)$ are within **27** of $E_6$, as can be seen by decomposing this under SL(6) and identifying them to lie within **15** of SL(6).

The generalization of the result to SL$(5,\mathbb{Z})$ $U$-duality (in $D=7$) is also straightforward. One can analogously consider the case of periodic network lattice involving 6-string junctions (as well as other similar structures) and define a set of five vectors, $\vec{\tilde{A}}_I$ $(I=1,\ldots,5)$ similar to $\vec{a},\vec{b},\vec{c}$ defined earlier. Similarly one has a set of other five vectors, $\vec{\tilde{V}}_I$, whose components are given in terms of quantum numbers $p_I,q_I,r_I,\ldots$ etc., as well as SL(5)/SO(5) moduli. The final mass formula, now with 1/32 supersymmetry has a form:

$$M^2_{BPS}=\left(\vec{\tilde{V}}_I\cdot\vec{\tilde{V}}_J\right)\left(\vec{\tilde{A}}_I\cdot\vec{\tilde{A}}_J\right)+\left(\vec{\tilde{V}}_I\times\vec{\tilde{V}}_J\right)\cdot\left(\vec{\tilde{A}}_I\times\vec{\tilde{A}}_J\right). \tag{21}$$

To conclude the discussion of the compactified non-planar networks as lower dimensional BPS states, we like to point out that several other possibilities of network compactification can be discussed by restricting to smaller subgroups of $U$-duality. For example, one can construct planar periodic networks of $(p,q,r)$-strings in eight dimensions, by considering SL$(2,\mathbb{Z})$ subgroups of SL$(3,\mathbb{Z})$. One then has 1/4 supersymmetric BPS states in six dimensions after compactifying these networks on $T^2$. It however remains to be seen whether one can obtain complete multiplets of the full duality symmetry, by combining various such possibilities of compactified networks.

We now discuss the application of the results to certain world-volume theories of branes, following a similar exercise for the case of planar IIB string networks [9, 10]. The planar IIB configurations are of interest from the point of view of 1/4 BPS dyon solutions of $N=4$ gauge theory. These $N=4$ theories in turn are considered to be the linearized approximation of the world-volume theories of D3-branes that are invariant under the SL$(2,\mathbb{Z})$ duality of the IIB theory. Moreover electric and magnetic charges also transform under this SL$(2,\mathbb{Z})$. In eight dimensional type-II theories, a similar role is played by 2-branes which are invariant under SL$(3,\mathbb{Z})$ and are known as $U2$-branes [15]. From the point of view of branes, this SL(3) acts on charges originating from three different components of $N=1, D=10$ gauge fields defining the world-volume theory. As an example, such charges can be identified in a 2-brane of this type by compactifying D4-branes on $T^2$. The SL$(3,\mathbb{Z})$ then acts on three charges, originating from the two internal components of the D4-brane gauge fields $(A_3,A_4)$ and a third one obtained by a Hodge-dualization of the three-dimensional gauge fields $A_\mu$ [17]. In a theory of parallel multi-branes, these fields are expected to form appropriate adjoint representations of the enhanced symmetries. Existence of the SL$(3,\mathbb{Z})$ symmetry on the world-volume can also be argued from the point of view of heterotic strings in $D=3$. The full duality symmetry of heterotic strings in $D=3$ is known to be $O(8,24,\mathbb{Z})$ [18]. The above SL$(3,\mathbb{Z})$, in this picture, then belongs to the SL$(8,\mathbb{Z})$ subgroup of $O(8,24,\mathbb{Z})$ , which transforms various components of ten-dimensional gauge fields, once again after Hodge-dualizations, in vector representations.

Then, to generalize the results of [9] we consider a configuration of four such branes and above configuration of 4-string junction is formed by strings ending on these $U2$-branes. For example, in figure $1a$, points $(A, B, C, D)$ can be identified with the positions of these branes. The world-volume of the branes is orthogonal to the three dimensional space of strings and junctions. Vectors $\vec{a}, \vec{b}, \vec{c}$ described above parameterize the vacuum expectation values of the adjoint Higgs fields in the resulting $N = 8$ supersymmetric theory. Now, to give a mass formula for such states while making connection with the work of [9], we choose special values for $p_I, q_I, r_I$ to be: $(p_1, q_1, r_1) = (1, 0, 0)$, $(p_2, q_2, r_2) = (0, 1, 0)$, $(p_3, q_3, r_3) = (0, 0, 1)$. In this case, BPS mass formula (10) reduces to

$$\begin{aligned} M^2 &= e^{-(\phi+\alpha)}|\vec{a}|^2 + e^{\alpha}|\vec{b}|^2 + e^{\phi}|\vec{c}|^2 + \\ &\quad + 2\left[(\vec{a}\times\vec{b})\cdot\hat{e}_3 e^{-\phi/2} + (\vec{a}\times\vec{c})\cdot\hat{e}_2 e^{-\alpha/2} + (\vec{b}\times\vec{c})\cdot\hat{e}_1 e^{(\phi+\alpha)/2}\right]. \end{aligned} \tag{22}$$

Following [9], we now interpret this as the mass of a 1/8 supersymmetric bound state in the world-volume theory described above. For this we define charges, similar to those in [9]:

$$\vec{Q}_1 = e^{-(\phi+\alpha)/2}\vec{a}\,, \qquad \vec{Q}_2 = e^{\alpha/2}\vec{b}\,, \qquad \vec{Q}_3 = e^{\phi/2}\vec{c}\,. \tag{23}$$

Since the role of the couplings on the world-volume theory is played by the space-time moduli $\phi, \alpha$ etc., and $(\vec{a}, \vec{b}, \vec{c})$ define the vacuum expectation values of scalars in this world-volume theory, $\vec{Q}_i$'s above can be identified with the physical charges in this theory. The subscripts in expression of charges now have their origins in fields $A_4, A_5, A_\mu$ mentioned above, whereas vector sign above them can be interpreted to be along certain $R$-symmetry directions. Similar form of energy expressions, for 1/2 BPS states involving these fields, were observed in [17]. However it is of interest to verify these results directly from the point of view of world-volume gauge theories, following a similar exercise in $D = 4$ in [19] and to further examine the properties of such bound states.

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